Gauss Project HWU17: Simulations of charm quarks: decoupling at low energies and charmonium

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We study the effects of a dynamical charm quark in lattice quantum chromodynamics. To this end we simulate a model consisting of $N_{\rm f}=2$ dynamical heavy quarks whose mass M reaches up to the charm quark mass. At energies much below M, the heavy quarks decouple. Their effects can be described by an effective theory with the heavy quarks removed. We compare the simulation results with predictions of the effective theory. Sub-percent precision is required to resolve the effects of a dynamical charm quark in low energy observables. We also study another class of observables containing valence charm quarks. Here the focus is on the control of cut-off effects which are known to be large in quantities like the hyperfine splitting. We perform direct simulations at the charm quark mass for very small values of the lattice spacing ($a=0.036\,{\rm fm}$ and $a=0.023\,{\rm fm}$). We present first results on the spectrum of charmonium.

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Upshot

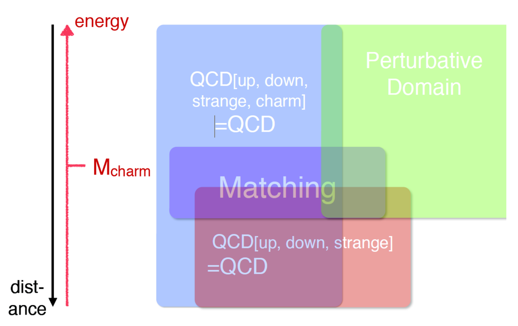
At present most simulations of lattice QCD are done with $N_{\rm f}=2+1$ dynamical quarks (up, down and strange). The inclusion of heavy quarks (charm) requires

- high precision in low energy observables to resolve tiny charm loop effects
- small lattice spacings to control cut-off effects proportional to the quark mass

Our project aims at estimating these effects. Since a comparison of $N_{\rm f}=2+1$ and $N_{\rm f}=2+1+1$ simulations is obscured by many uncertainties, we study a model, namely QCD with two heavy, mass-degenerate quarks.

Effective theory

Consider QCD with $N_{\rm q}$ quarks. $\Lambda_{\rm q}$ is the Lambda parameter in the $\overline{\rm MS}$ scheme. $N_{\rm l}$ quarks are light and we set their mass to zero. $N_{\rm q}-N_{\rm l}$ quarks are heavy and their renormalization group invariant (RGI) mass is M.



Effective theory for energies $E \ll M$, defined in terms of $N_{\rm l}$ light quarks with Lagrangian [Weinberg, Physica **A96** (1979)]

$$\mathcal{L}_{\rm dec} = \mathcal{L}_{{\rm QCD}_{N_{\rm l}}} + (1/M)^2 \sum_i \omega_i \Phi_i + {\rm O}((\Lambda_{\rm q}/M)^4)\,.$$

Φ_i are fields of dimension 6 and ω_i are dimensionless parameters.

Decoupling

At leading order the effective theory is QCD with $N_{\rm l}$ massless quarks. QCD$_{N_{\rm l}}$ has only one free parameter, the gauge coupling $\bar{g}_{\rm l}(\mu/\Lambda_{\rm l})$. One can specify either a value for the coupling at some scale μ or equivalently the Lambda parameter. Matching yields

$$\Lambda_{\rm l} = \Lambda_{\rm dec}(M,\Lambda_{\rm q})$$

which determines $\Lambda_{\rm l}$ as a function of M and $\Lambda_{\rm q}$.

$m^{\rm had}$ denotes a hadron mass or a hadronic scale like $1/\sqrt{t_0}$ [Lüscher, arXiv:1006.4518]. The non-perturbative matching condition is

$$m^{\rm had}_{\rm q} = m^{\rm had}_{\rm l} + {\rm O}((\Lambda_{\rm q}/M)^2)\,,$$

where $m^{\rm had}_{\rm q}$ ($m^{\rm had}_{\rm l}$) is the hadron mass computed in QCD$_{N_{\rm q}}$ (QCD$_{N_{\rm l}}$). A consequence of the matching is the factorisation [Bruno, Finkenrath, Knechtli, Leder and Sommer, PRL **114**, 102001 (2015)]

$$\frac{m^{\rm had}_{\rm q}(M)}{m^{\rm had}_{\rm q}(0)} = Q^{\rm had}_{\rm l,q} \times P_{\rm l,q}(M/\Lambda_{\rm q}) + {\rm O}((\Lambda_{\rm q}/M)^2)\,,$$

$$Q^{\rm had}_{\rm l,q} = \frac{m^{\rm had}_{\rm l}/\Lambda_{\rm l}}{m^{\rm had}_{\rm q}(0)/\Lambda_{\rm q}}\,, \qquad P_{\rm l,q}(M/\Lambda_{\rm q}) = \frac{\Lambda_{\rm l}}{\Lambda_{\rm q}}\,.$$

The factor $Q^{\rm had}_{\rm l,q}$ is non-perturbative and independent of M. It depends on the hadronic scale. The factor $P_{\rm l,q}$ can be computed in perturbation theory and depends on M. It does not depend on the hadronic scale.

Simulations

We simulate a theory with $N_{\rm q}=2$ heavy quarks and compare it to Yang-Mills theory ($N_{\rm l}=0$). We use the publicly available openQCD package [Lüscher and Schaefer, arXiv:1206.280; http://luscher.web.cern.ch/luscher/openQCD/]

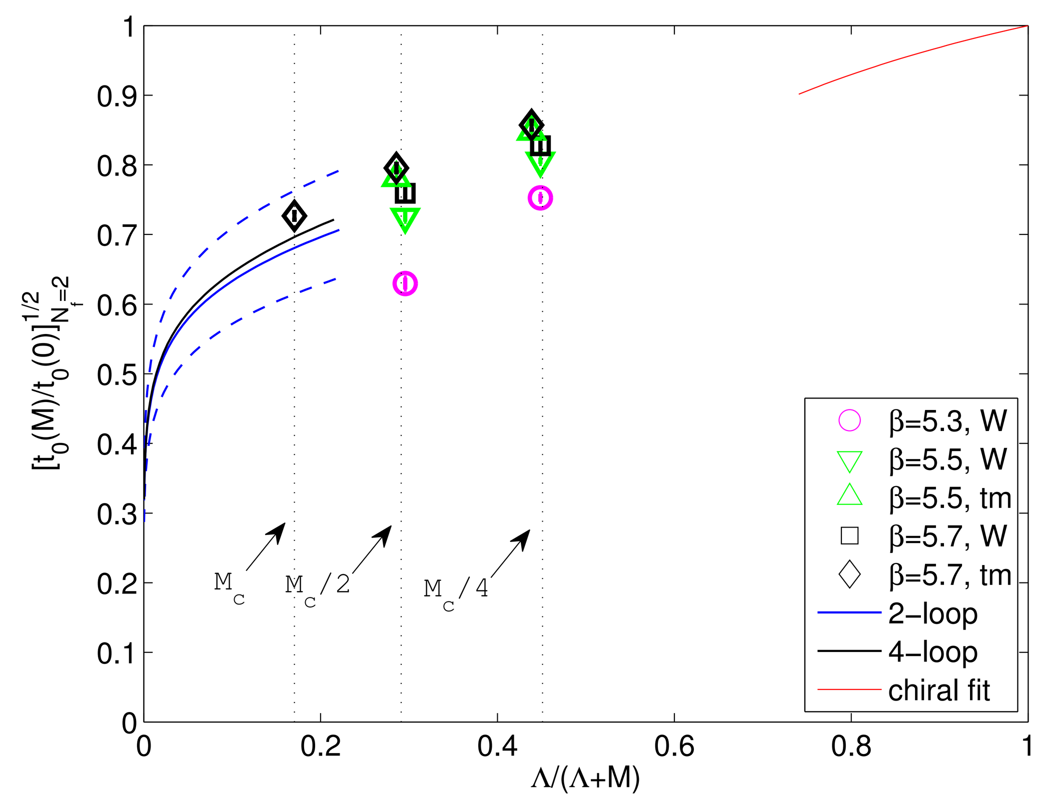
β	a [fm]	A	BC	$T\times L^3$	$M/\Lambda_{\overline{\rm MS}}$	kMDU	$\tau_{\rm exp}$	M
5.3	0.0658(10)	W	p	64×32^3	0.638(46)	1.0	0.07	DESY
			p	64×32^3	1.308(95)	2.0	0.05	DESY
			p	64×32^3	2.60(19)	2.0	0.04	DESY
		tm	o	120×32^3	0.59	4.3	0.08	HLRN
5.5	0.0486(7)	W	o	120×32^3	0.630(46)	8.5	0.15	HLRN
			o	120×32^3	1.282(93)	8.1	0.12	HLRN
			p	96×48^3	2.45(18)	4.0	0.10	CHEOPS
		tm	o	120×32^3	0.59	8.0	0.16	HLRN
			o	120×32^3	1.28	8.0	0.14	HLRN
			o	120×32^3	2.50	8.0	0.12	HLRN
5.7	0.0358(6)	W	o	192×48^3	0.587(43)	4.0	0.28	J
			o	192×48^3	1.277(94)	4.2	0.24	J
			o	192×48^3	2.50(18)	8.5	0.20	J
		tm	o	192×48^3	0.59	4.0	0.29	J
			o	192×48^3	1.28	16.2	0.25	J
			o	192×48^3	2.50	9.0	0.22	J
			o	120×32^3	4.87	18.0	0.18	HLRN
6.0	0.0229(4)	tm	o	192×48^3	4.87	17.6	0.43	J

Mass-dependence of $\sqrt{t_0(M)/t_0(0)}$

The factorisation formula for the scale $1/\sqrt{t_0}$ predicts

$$\sqrt{t_0(M)/t_0(0)}_{N_{\rm q}=2} = 1/(P_{0,2} Q^{\sqrt{t_0}}_{0,2}) + {\rm O}((\Lambda/M)^2)\,,$$

where $Q^{\sqrt{t_0}}_{0,2} \simeq 1.19(13)$ [ALPHA; Sommer, arXiv:1401.3270]



We simulate $N_{\rm q}=2$ O(a) improved Wilson quarks and twisted mass quarks at maximal twist with plaquette gauge action. We use the non-perturbative clover coefficient [Jansen and Sommer, hep-lat/9803017]. The mass-dependence of $\sqrt{t_0(M)/t_0(0)}$ is well described by the factorisation formula at the level of 10% precision.

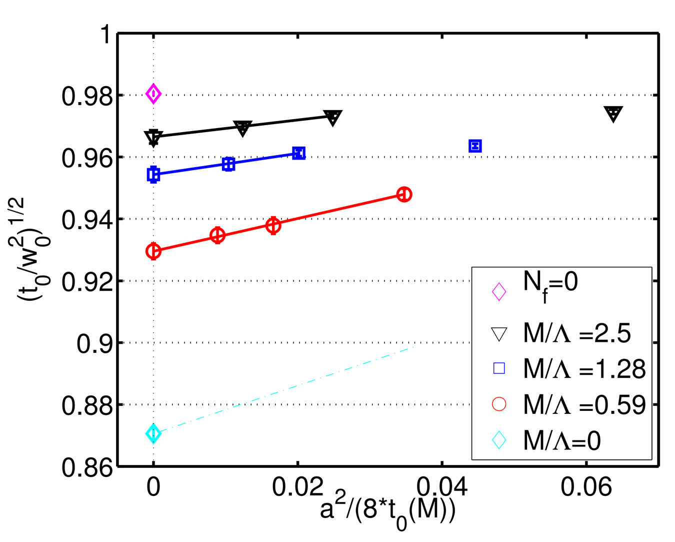
(Wilson data are slightly displaced horizontally for clarity)

Charm effects at low energies

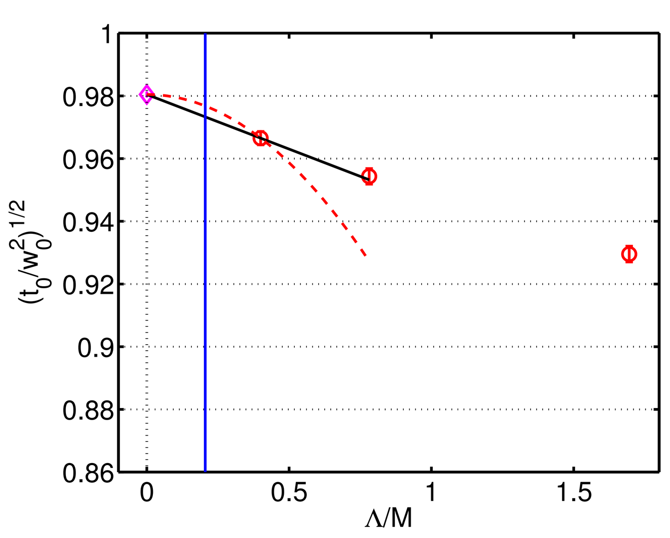
Ratios of hadronic scales are independent of Λ, factor P drops out. Power corrections

$$R(M) = \left.\frac{\sqrt{t_0(M)}}{w_0(M)}\right|_{N_{\rm q}=2,N_{\rm l}=0} = \left.\frac{\sqrt{t_0}}{w_0}\right|_{N_{\rm l}=0} + {\rm O}((\Lambda/M)^2)\,,$$

where w_0 is defined in [Borsanyi et al., arXiv:1203.4469].



Continuum extrapolation of $R=\sqrt{t_0/w_0^2}$

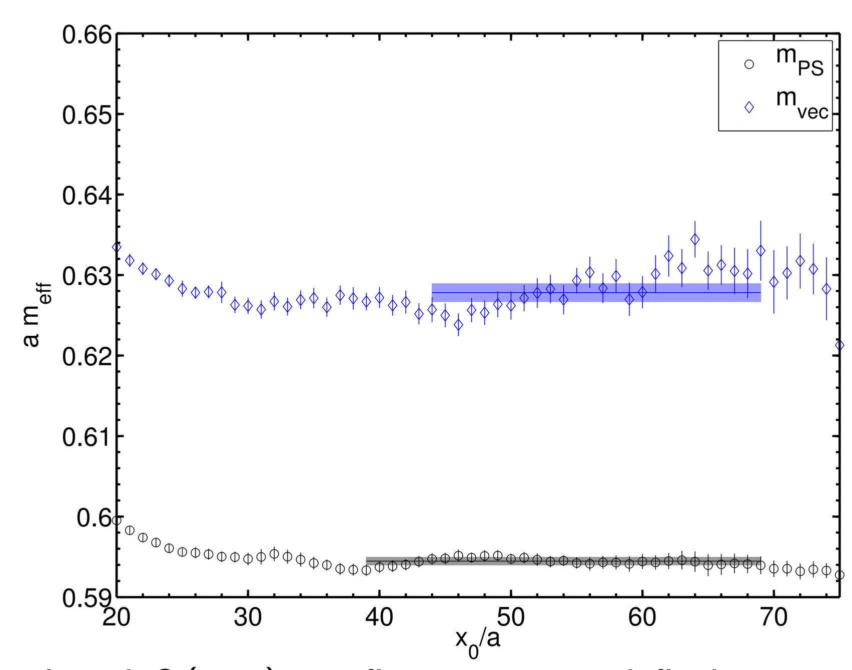


Mass-dependence in the continuum

The relative effects rescaled to a single heavy quark ($N_{\rm q}=2$) are $\frac{1}{N_{\rm q}}\frac{R(M)-R(\infty)}{R(\infty)}$. Interpolation to the charm quark mass $M_c/\Lambda=4.87$ yields 0.37(5)% ($1/M$-scaled) or 0.19(3)% ($1/M^2$-scaled). Sub-percent precision is required to resolve charm loop effects

Charmonium (FK, SC, TK, BL, GM)

Hyperfine splitting $m_{J/\psi}-m_{\eta_c}=113.3\,{\rm MeV}$ is a quantity which exhibits large cut-off effects, cf. [Cho et al., arXiv:1504.01630; Rae and Dürr, arXiv:1509.02381]. We simulate a doublet (c c') of twisted mass quarks at lattice spacings $a=0.036\,{\rm fm}$ and $a=0.023\,{\rm fm}$. We use $\bar{c}\Gamma c'$ operators, in the physical basis: pseudo-scalar $\Gamma_{PS}=\gamma_5$ and vector $\Gamma_V=\gamma_i$



At $a=0.036\,{\rm fm}$ we analysed O(500) configurations and find $r=m_V/m_{PS}=1.056(2)$; cf. [Kalinowski and Wagner, arXiv:1509.02396], who find $r=1.060$ with $N_{\rm f}=2+1+1$ twisted mass quarks at $a=0.089\,{\rm fm}$. Difference with physical value $r=1.038$ due to disconnected contributions and cut-off effects ⇒ goal of future work is to disentagle these effects and to investigate the effects of charm loops on observables with explicit charm quarks